“Solving Equations with Algebra Tiles and Expression Mats”

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2.1.1 What is a variable?

Exploring Variables and Expressions

In Algebra and in future mathematics courses, you will work with unknown quantities that can be represented using variables. Today manipulatives called “algebra tiles” will be introduced to help you and your teammates answer some important questions, such as “What is a variable?” and “How can we use it?”

2-1. Your teacher will distribute a set of algebra tiles for your team to use during this course. As you explore the tiles, address the following questions with your team. Be prepared to share your responses with the class.

- How many different shapes are there? What are all of the different shapes?
- How are the shapes different? How are they the same?
- How are the shapes related? Which fit together and which do not?

2-3. JUMBLED PILES

Your teacher will show you a jumbled pile of algebra tiles and will challenge you to write a name for the collection. What is the best description for the collection of tiles? Is your description the best possible?
2.1.2 What is the perimeter?

Simplifying Expressions by Combining Like Terms

While Lesson 2.1.1 focused on the area of algebra tiles, today’s lesson will focus on the perimeter. What is perimeter? How can you find it? By the end of this lesson, you will be able to find the perimeter of complex shapes formed by collections of tiles.

Your class will also focus on several ways to find perimeter, recognizing that there are different ways to “see” or recognize perimeter. Sometimes, with complex shapes, a convenient shortcut can help you find the perimeter more quickly. Be sure to share any insight into finding perimeter with your teammates and with the whole class.

2-11. Your teacher will provide a set of algebra tiles for your team to use today. Separate one of each shape and review its name (area). Then find the perimeter of each tile. Decide with your team how to write a simplified expression that represents the perimeter of each tile. Be prepared to share the perimeters you find with the class.

2-13. For each of the shapes formed by algebra tiles below:

- Use tiles to build the shape.
- Sketch and label the shape on your paper. Then write an expression that represents the perimeter.
- Simplify your perimeter expression as much as possible. This process of writing the expression more simply by collecting together the parts of the expression that are the same is called combining like terms.

2-14. Calculate the perimeter of the shapes in problem 2-13 if the length of each x-tile is 3 units and the length of each y-tile is 8 units. Show all work.
2.1.3 What does “minus” mean?

Writing Algebraic Expressions

In this section, you will look at algebraic expressions and see how they can be interpreted using an Expression Mat. To achieve this goal, you first need to understand the different meanings of the “minus” symbol, which is found in expressions such as $5 - 2$, $-x$, and $-(-3)$.

2-22. LEARNING LOG

What does “−” mean? Find as many ways as you can to describe this symbol and discuss how these descriptions differ from one another. Share your ideas with the class and record the different uses in your Learning Log. Title this entry “Meanings of Minus” and include today’s date.

2-23. USING AN EXPRESSION MAT

Your introduction to algebra tiles in Lessons 2.1.1 and 2.1.2 involved only positive values. Today you will look at how you can use algebra tiles to represent “minus.” Below are several tiles with their associated values. Note that the shaded tiles are positive and the unshaded tiles are negative. The diagram at right will appear throughout the text as a reminder.

\[
\begin{align*}
3 & = 5 \\
-3 & = 3x \\
-2y & = -2y
\end{align*}
\]

“Minus” can also be represented with a new tool called an Expression Mat, shown at right. An Expression Mat is an organizing tool that will be used to represent expressions. Notice that there is a positive region at the top and a negative (or “opposite”) region at the bottom.

Using the Expression Mat, the value $-3$ can be shown in several ways, two of which are shown at right.

Note that in these examples, the diagram on the left side uses negative tiles in the “+” region, while the diagram on the right side uses positive tiles in the “−” region.
a. Build two different representations for \(-2x\) using an Expression Mat.

b. Similarly, build \(3x - (-4)\). How many different ways can you build \(3x - (-4)\)?

2-24. As you solved problem 2-23, did you see all of the different ways to represent “minus” that you listed in problem 2-22? Discuss how you could use an Expression Mat to represent the different meanings discussed in class.

2-25. BUILDING EXPRESSIONS

Use the Expression Mat to create each of the following expressions with algebra tiles. Find at least two different representations for each expression. Sketch each representation on your paper. Be prepared to share your different representations with the class.

a. \(-3x + 4\)  
b. \(-(y - 2)\)  
c. \(-y - 3\)  
d. \(5x - (3 - 2x)\)

2-26. In problem 2-25, you represented algebraic expressions with algebra tiles. In this problem, you will need to reverse your thinking to write an expression from a diagram of algebra tiles.

Working with a partner, write algebraic expressions for each representation below. Start by building each problem using your algebra tiles.

a. 

b. 

c. 

d. 

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2.1.4  What makes zero?

Using Zero to Simplify Algebraic Expressions

Today you will continue your work with rewriting algebraic expressions. As you work with your team, ask yourself and your teammates these focus questions:

How did you see it?
How can you write it?
Is your expression as simplified as possible?

2-35. How can you represent zero with tiles on an Expression Mat? With your team, try to find at least two different ways to do this (and more if you can). Be ready to share your ideas with the class.

2-36. Gretchen used seven algebra tiles to build the expression shown below.

   a. Build this collection of tiles on your own Expression Mat and write its value.
   b. Represent this same value three different ways, each time using a different number of tiles. Be ready to share your representations with the class.

2-37. Build each expression below so that your representation does not match those of your teammates. Once your team is convinced that together you have found four different, valid representations, sketch your representation on your paper and be ready to share your answer with the class.

   a. \(-3x + 5 + y\)  
   b. \(-(-2y + 1)\)  
   c. \(2x - (x - 4)\)

2-38. Write the algebraic expression shown on each Expression Mat below. Build the model and then simplify the expression by removing as many tiles as you can without changing the value of the expression. Finally, write the simplified algebraic expression.

   a. 
   b.
How can I simplify the expression?

Using Algebra Tiles to Simplify Algebraic Expressions

Which is greater: 58 or 62? That question might seem easy, because the numbers are ready to be compared. However, if you are asked which is greater, \(2x + 8 - x - 3\) or \(6 + x + 1\), the answer is not so obvious! In this lesson, you and your teammates will investigate how to compare two algebraic expressions and decide whether one is greater.

2-46. For each expression below:
- Use an Expression Mat to build the expression.
- Find a different way to represent the same expression using tiles.

a. \(7x - 3\)  
b. \(-2x + 6 + 3x\)

2-47. COMPARING EXPRESSIONS

Two expressions can be represented at the same time using an Expression Comparison Mat. The Expression Comparison Mat puts two Expression Mats side-by-side so you can compare them and see which one is greater. For example, in the picture at right, the expression on the left represents \(-3\), while the expression on the right represents \(-2\). Since \(-2 > -3\), the expression on the right is greater.

Build the Expression Comparison Mat shown at right. Write an expression representing each side of the Expression Mat.

a. Can you simplify each of the expressions so that fewer tiles are used? Develop a method to simplify both sides of the Expression Comparison Mats. Why does it work? Be prepared to justify your method to the class.

b. Which side of the Expression Comparison Mat do you think is greater (has the largest value)? Agree on an answer as a team. Make sure each person in your team is ready to justify your conclusion to the class.
2-48. As Karl simplified some algebraic expressions, he recorded his work on the diagrams below.

- Explain in writing what he did to each Expression Comparison Mat on the left to get the Expression Comparison Mat on the right.
- If necessary, simplify further to determine which Expression Mat is greater. How can you tell if your final answer is correct?

a. ![Diagram A]

b. ![Diagram B]

c. ![Diagram C]

2-49. Use Karl’s “legal” simplification moves to determine which side of each Expression Comparison Mat below is greater. Record each of your “legal” moves on the Lesson 2.1.5A Resource Page by drawing on it the way Karl did in problem 2-49. After each expression is simplified, state which side is greater (has the largest value). Be prepared to share your process and reasoning with the class.

a. ![Diagram D]

b. ![Diagram E]
2.1.6 Which is greater?

Using Algebra Tiles to Compare Expressions

Can you always tell whether one algebraic expression is greater than another? In this lesson, you will compare the values of two expressions, practicing the different simplification strategies you have learned so far.

2-56. WHICH IS GREATER?

Write an algebraic expression for each side of the Expression Comparison Mats given below. Use the “legal” simplification moves you worked with in Lesson 2.1.5 to determine which expression on the Expression Comparison Mat is greater.

a. 

\[
\begin{array}{c|c}
\text{Left} & \text{Right} \\
\hline
\text{Which is greater?} & \text{Which is greater?} \\
\end{array}
\]

b. 

\[
\begin{array}{c|c}
\text{Left} & \text{Right} \\
\hline
\text{Which is greater?} & \text{Which is greater?} \\
\end{array}
\]

e. 

\[
\begin{array}{c|c}
\text{Left} & \text{Right} \\
\hline
\text{Which is greater?} & \text{Which is greater?} \\
\end{array}
\]

f. 

\[
\begin{array}{c|c}
\text{Left} & \text{Right} \\
\hline
\text{Which is greater?} & \text{Which is greater?} \\
\end{array}
\]

2-57. Build the Expression Comparison Mat shown below with algebra tiles.

a. Simplify the expressions using the “legal” moves that you developed in Lesson 2.1.5.

b. Can you tell which expression is greater? Explain in a few sentences on your paper. Be prepared to share your conclusion with the class.
2.1.7 How can I write it?

Simplifying and Recording Work

Today you will continue to compare expressions as you strengthen your simplification strategies. At the same time, you will work with your class to find ways to record your work so that another student can follow your strategies.

2-63. Use algebra tiles to build the expressions below on an Expression Comparison Mat. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to tell which expression is greater, explain why.

a. Which is greater: $3x - (2 - x) + 1$ or $-5 + 4x + 4$?

b. Which is greater: $2x^2 - 2x + 6 - (-3x)$ or $-(3 - 2x^2) + 5 + 2x$?

c. Which is greater: $-1 + 6y - 2 + 4x - 2y$ or $x + 5y - (-2 + y) + 3x - 6$?

2-64. RECORDING YOUR WORK

Although using algebra tiles can make some things easier because you can “see” and “touch” the math, it can be difficult to remember what you did to solve a problem unless you take good notes.

Use the simplification strategies you have learned to determine which expression on the Expression Comparison Mat at right is greater. Record each step as instructed by your teacher. Also record the simplified expression that remains after each move. This will be a written record of how you solved this problem. Discuss with your team what the best way is to record your moves.
2-65. While Athena was comparing the expressions shown at right, she was called out of the classroom. When her teammates needed help, they looked at her paper and saw the work shown below. Unfortunately, she had forgotten to explain her simplification steps.

Can you help them figure out what Athena did to get each new set of expressions?

<table>
<thead>
<tr>
<th>Left Expression</th>
<th>Right Expression</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x + 4 - x - (-2) + x^2$</td>
<td>$-1 + x^2 + 4x - (4 + 2x)$</td>
<td>Original expressions</td>
</tr>
<tr>
<td>$3x + 4 - x - (-2)$</td>
<td>$-1 + 4x - (4 + 2x)$</td>
<td></td>
</tr>
<tr>
<td>$3x + 4 - x + 2$</td>
<td>$-1 + 4x - 4 - 2x$</td>
<td></td>
</tr>
<tr>
<td>$2x + 6$</td>
<td>$2x - 5$</td>
<td></td>
</tr>
<tr>
<td>$6$</td>
<td>$-5$</td>
<td>Because $6 &gt; -5$, the left side is greater.</td>
</tr>
</tbody>
</table>

2-66. For each pair of expressions below, determine which is greater, carefully recording your steps as you go. If you cannot tell which expression is greater, state, “Not enough information.” Make sure that you record your result after each type of simplification. For example, if you flip all of the tiles from the “−” region to the “+” region, record the resulting expression and indicate what you did using either words or symbols. Be ready to share your work with the class.

a. Which is greater: $3x^2 - y^2 + x - 2$ or $-1 + y^2 + x - 2$?

b. Which is greater: $x^2 - y^2 + x - 2$ or $-1 + y^2 + x - 2$?

c. Which is greater: $5 - (2y - 4) - 2$ or $-y - (1 + y) + 4$?

d. Which is greater: $3xy + 9 - 4x - 7 + x$ or $-2x + 3xy - (x - 2)$?
2.1.8 What if both sides are equal?

Using Algebra Tiles to Solve for $x$

Can you always tell whether one algebraic expression is greater than another? In this section, you will continue to practice the different simplification strategies you have learned so far to compare two expressions and see which one is greater. However, sometimes you do not have enough information about the expressions. When both sides of an equation are equal, you can learn even more about $x$. As you work today, focus on these questions:

How can you simplify?

How can you get $x$ alone?

Is there more than one way to simplify?

Is there always a solution?

2-72. WHICH IS GREATER?

Build each expression represented below with the tiles provided by your teacher. Use “legal” simplification moves to determine which expression is greater, if possible. If it is not possible to determine which expression is greater, explain why it is impossible. Be sure to record your work on your paper.

a. Left
   ![Image of algebra tiles]
   Which is greater?

   Right
   ![Image of algebra tiles]
   Which is greater?

   a. Which is greater:
   $x + 1 - (1 - 2x)$ or $3 + x - 1 - (x - 4)$?

2-73. WHAT IF BOTH SIDES ARE EQUAL?

If the number 5 is compared to the number 7, then it is clear that 7 is greater. However, what if you compare $x$ with 7? In this case, $x$ could be smaller, larger, or equal to 7.

Examine the Expression Comparison Mat below.

a. If the left expression is smaller than the right expression, what does that tell you about the value of $x$?
2-73.  *problem continued from previous page*

b.  If the left expression is greater than the right expression, what does that tell you about the value of $x$?

c.  What if the left expression is equal to the right expression?  What does $x$ have to be for the two expressions to be equal?

2-74.  **SOLVING FOR $x$**

In later courses, you will learn more about situations like parts (a) and (b) in the preceding problem, called “inequalities.”  For now, to learn more about $x$, assume that the left expression and the right expression are equal.  The two expressions will be brought together on one mat to create an Equation Mat, as shown in the figure below.  The double line down the center of an Equation Mat represents the word “equals.”  It is a wall that separates the left side of an equation from the right side.

a.  Obtain the “Equation Mat” resource page from your teacher.  Build the equation represented by the Equation Mat at right using algebra tiles.  Simplify as much as possible and then solve for $x$.  Be sure to record your work.

b.  Build the equation $2x - 5 = -1 + 5x + 2$ using your tiles by placing $2x - 5$ on the left side and $-1 + 5x + 2$ on the right side.  Then use your simplification skills to simplify this equation as much as possible so that $x$ is alone on one side of the equation.  Use the fact that both sides are equal to solve for $x$.  Record your work.
Expression Mat
Lesson 2.1.5A Resource Page
Problem 2-46

a. Left

b. Left

Which is greater?

= +1
= -1

Which is greater?

= +1
= -1

Which is greater?

= +1
= -1

Which is greater?

= +1
= -1
Expression Comparison Mat

Which is greater?

Left

Which is greater?

Right
Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy).

1. Make sense of problems and persevere in solving them.
   - Find meaning in problems
   - Look for entry points
   - Analyze, conjecture and plan solution pathways
   - Monitor and adjust
   - Verify answers
   - Ask themselves the question: "Does this make sense?"

2. Reason abstractly and quantitatively.
   - Make sense of quantities and their relationships in problems
   - Learn to contextualize and decontextualize
   - Create coherent representations of problems

3. Construct viable arguments and critique the reasoning of others.
   - Understand and use information to construct arguments
   - Make and explore the truth of conjectures
   - Recognize and use counterexamples
   - Justify conclusions and respond to arguments of others

4. Model with mathematics.
   - Apply mathematics to problems in everyday life
   - Make assumptions and approximations to simplify a complicated situation
   - Identify quantities in a practical situation
   - Interpret results in the context of the situation and reflect on whether the results make sense

5. Use appropriate tools strategically.
   - Consider the available tools when solving problems
   - Are familiar with tools appropriate for their grade or course (pencil and paper, concrete models, ruler, protractor, calculator, spreadsheet, computer programs, digital content located on a website, and other technological tools)
   - Make sound decisions of which of these tools might be helpful

6. Attend to precision.
   - Communicate precisely to others
   - Use clear definitions, state the meaning of symbols and are careful about specifying units of measure and labeling axes
   - Calculate accurately and efficiently

7. Look for and make use of structure.
   - Discern patterns and structures
   - Can step back for an overview and shift perspective
   - See complicated things as single objects or as being composed of several objects

8. Look for and express regularity in repeated reasoning.
   - Notice if calculations are repeated and look both for general methods and shortcuts
   - In solving problems, maintain oversight of the process while attending to detail
   - Evaluate the reasonableness of their immediate results